Event-Triggering Containment Control for a Class of Multi-Agent Networks With Fixed and Switching Topologies

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***Abstract*— This paper investigates the event-triggering containment control of multi-agent networks with fixed and switching topologies, where there exist interactions between the leaders. Two cases are studied, respectively: One is that the leaders are opinionated. Then each leader will converge to its initial goal and the followers converge to the convex hull formed by the leaders. The other is that the leaders are not opinionated. Then the leaders will not insist on their initial goals and converge to some compromising states due to the interactions among them, while the followers still converge to the convex hull formed by the leaders. By means of pull-based event-triggering control, the event-triggering containment control problems of multi-agent systems with fixed and switching topologies are investigated, respectively. Finally, simulations are given to illustrate the theoretical results.**

***Index Terms*— Containment control, event-triggering control, multi-agent systems, sampled-data control.**

I. Introduction

RECENTLY, collective behaviors of multi-agent systems have received increasing interests due to their broad applications in biological systems, Unmanned Air Vehicles, robotic systems and various other fields (see [1] and the references therein). In the past decade, plentiful results on cooperative control problems of multi-agent systems have been reported, e.g., consensus [2], [3], synchronization control [4], [5] and containment control [6]–[9]. During the past decade, containment control has arisen increasing research attention due to its wide applications. For instance, in UAV, a group of agents move from one target to another when only a portion of the agents is equipped with necessary sensors to detect the hazardous obstacles such that the agents who are not equipped will stay in a safety area formed by the equipped agents [8]. Thus, it is necessary to investigate the containment control problem of multi-agent systems with multiple leaders, which means that the followers will converge to the convex hull formed by the multiple leaders. In [7], a stop-and-go strategy was proposed to drive a collection of mobile agents to the convex polytope spanned by multiple stationary/moving leaders. The problem of containment control was investigated for a group of mobile autonomous agents with multiple stationary or dynamic leaders under both fixed and switching directed network topologies in [6]. It should be noted that, in the above mentioned works on containment control of multi-agent systems, it is assumed that there are no interactions between the leaders. However, in practice, the interactions among the leaders will influence our emotions, opinions, and behaviors [10], [11]. Usually, Due to the interactions among the leaders, there exist two cases for the leaders: 1) Leaders are opinionated: the leaders have their own decisions (goals) before discussion and they will insist on their original decisions (goals). For the followers, as they learn the information from the leaders through the communication graph, the followers will make their final decisions determined by the leaders and the communication topology. 2) Leaders are not opinionated: the leaders have their own decisions (goals) before discussion. Due to the interactions among the leaders, the leaders may have some compromises and they will make their decisions relying on other leaders' original goals and their own ones, while the followers still converge to the convex hull formed by the leaders. Therefore, it is of paramount importance to investigate the containment control of multi-agent systems, in which there are communication links among the leaders.

In the past years, some results have been reported on containment control of multi-agent systems [12], [7]–[8]. However, in [12], [7]–[8], there is a common assumption that the information of each agent can be transmitted through the communication network continuously. As shown in [13], communications in practical networks may be subjected to various constraints and thus the continuous communication

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is not practical. Nowadays, controllers are implemented on digital platforms equipped with small embedded microprocessors capable of running real-time operating systems, since digital control can reduce the deployment cost of complex control systems. Hence, in order to mitigate the communication resources, event-triggering control schemes have been proposed for numerous systems such as networked control systems [14]–[16], wireless networks [17] and multi-agent systems [18]–[22]. For instance, in [15], the event-triggering H_∞ control was investigated for discrete-time networked control systems, where a new random process was first developed to model the input data sequence of the controller under the effect of the buffer. Furthermore, in [16], a novel polynomial event-triggered scheme was first proposed to determine the transmission of the signal and then the event-triggering fault-detection problem was investigated.

On the other hand, there are some results on event-triggering containment control of multi-agent systems [23]–[25]. In [23]–[25], each node uses the information of its local neighborhood at their latest triggered instants, which can be regarded as a special kind of push-based event-triggering mechanisms [26]. It can be seen that when implementing this event-triggering mechanism, one needs to monitor whether the state of its neighbors is updated. In this paper, inspired by [26], a pull-based event-triggering mechanism will be used to investigate the containment control of multi-agent systems, where an agent does not update its control law when the state of its neighbors is updated. In addition, due to the finite interaction region of sensing devices, many dynamical systems may suffer from a few unpredictable structural changes, such as random failures and repairs of sudden environment disturbances [27]–[30]. For example, the authors in [27], [29] have pointed out that the switching phenomenon cannot be overlooked in networked control systems. Thus, the topology of networked multi-agent systems may exhibit switching phenomena and it is important to investigate the event-triggering containment control of multi-agent systems with switching topology. However, to the best of our knowledge, the event-triggering containment control for multi-agent systems with switching topology is still an open issue primarily due to the difficulties in handling the coexistence of multiple network topologies and event-triggering mechanisms.

Based on the above discussions, in this paper, we aim to investigate the containment control of multi-agent networks with fixed and switching topologies. Based on the pull-based event-triggering control, containment problems of multi-agent networks with fixed and switching topologies are investigated, respectively. The main contributions of this paper are listed as follows: 1) Compared with the well-studied results on containment control problem of multi-agent networks [6], [31], [7]–[8], an event-triggering protocol is presented to solve the containment control problem of multi-agent networks, where there exist interactions among the leaders. In addition to this difference, two cases are considered, respectively: one is that the leaders are opinionated and the other is that the leaders are not opinionated. 2) Compared with the existing results on event-triggering containment control problem of multi-agent networks [23]–[25], a pull-based event-triggering protocol is presented to solve the containment control problem of multi-agent networks with fixed and switching topologies, respectively, where each agent does not need to update its control law when the state of its neighbors is updated. In addition, the event-triggering containment control problem of multi-agent systems with switching topology is also investigated.

Notations: $\mathbb{N}$ denotes the natural number. For $x \in \mathbb{R}^n$, x^T denotes its transpose. The vector norm is defined as $\|x\| = \sqrt{x^T x}$. I_n denotes n-dimensional identity matrix. For matrix $A \in \mathbb{R}^{n\times n}$, $\|A\| = \sqrt{\lambda_{\max}(A^T A)}$, where $\lambda_{\max}(\cdot)$ represents the largest eigenvalue.

II. Preliminaries

In this section, some basic concepts and model formulation are introduced.

Consider a group of N agents given by

$$\dot{x}_i(t) = u_i(t), \quad i = 1, 2, \ldots, N \tag{1}$$

where $x_i(t) \in \mathbb{R}^n$ and $u_i(t) \in \mathbb{R}^n$ are, respectively, the position and the control input. Without loss of generality, in this paper, we suppose that the agents labeled by $1, 2, \ldots, N-m$ are followers and the other m agents are leaders. $\mathcal{L}$ and $\mathcal{F}$ denote, respectively the set of leaders and followers.

In the following, we firstly focus on the containment control problem of multi-agent networks with event-triggering control. Next, we will extend the result of containment control of multi-agent systems with fixed topology into the case of switching topology in Section III-B. The network topology of our considered multi-agent system with fixed topology is defined as follows: denote by $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathcal{A})$ the directed graph with order N, where $\mathcal{V} = \mathcal{F}\cup\mathcal{L}$ is the set of nodes, $\mathcal{E} = (i, j) : i, j \in \mathcal{V}$ is the set of edges, and $\mathcal{A} = (a_{ij})_{N\times N}$ is the weighted adjacency matrix. An edge of $\mathcal{G}$ is represented by a pair of distinct nodes $(j, i) \in \mathcal{E}$, where node i can receive information from node j. $N_i = \{j \in \mathcal{V} | (j, i) \in \mathcal{E}\}$ denotes the set of neighbors of node i and $|N_i|$ stands for the cardinality of N_i. A directed path in a graph is a sequence $i_0, i_1, \ldots, i_l$ of distinct nodes such that $(i_{l_1}, i_{l_1-1}) \in \mathcal{E}$, $l_1 = 1, 2, \ldots, l$. A directed graph has a directed spanning tree if there exists at least one agent that has a directed path to every other agent. The in-degree of node i is defined as $d_i = \sum_{j\in N_i} a_{ij}$ and the diagonal in-degree matrix is defined as $\mathcal{D} = \mathrm{diag}\{d_1, d_2, \ldots, d_N\}$, $i = 1, 2, \ldots, N$. The Laplacian matrix of the weighted graph $\mathcal{G}$ is defined as $L = \mathcal{D} - \mathcal{A}$. It is well-known that L has at least one zero eigenvalue with a corresponding eigenvector $\mathbf{1}_N$, where $\mathbf{1}_N$ is an $N \times 1$ all-one column vector.

Consider the following pull-based event-triggering protocol with fixed topology: when $t \in [t_k^i, t_{k+1}^i)$

$$u_i(t) = \beta \sum_{j=N-m+1}^{N} a_{ij}(\delta_i(t_k^i) - \delta_j(t_k^i)) + \gamma\,\delta_i(t_k^i), i \in \mathcal{L},$$

$$u_i(t) = -h \sum_{j=1}^{N} a_{ij}(x_i(t_k^i) - x_j(t_k^i)), i \in \mathcal{F} \tag{2}$$

where $\gamma \geq 0$, $\beta > 0$, $h > 0$ are the control gains. a_{ij} is the element of the interaction topology $\mathcal{G}$. $\delta_i(t) = r_i - x_i(t)$ and

r_i is the desired objective state. t_k^i ($t_0^i = 0$) is the triggering instant to be determined according to the following event-triggering function:

$$t_{k+1}^i = \inf\{t > t_k^i : f_i > 0\},\ k \in \mathbb{N}, \tag{3}$$

where f_i is the triggering function that will be given in the end of this section, as some variables in f_i will be provided later on.

Remark 1: Recently, event-triggering containment control problems of multi-agent systems have been investigated in [23]–[25]. Compared with these results, our results have some differences in both network topology and event-triggering strategy:

1) *The difference in network topology*: In [23]–[25], the event-triggering containment control problem was investigated for a class of multi-agent systems with fixed topology. However, in this paper, both fixed and switching topologies are considered, which means that the topology considered in our paper is more general than the one in [23]–[25]. In addition to this major difference, two different cases are considered here, respectively: one is that the leaders are opinionated and the other is that the leaders are not opinionated.

2) *The difference in strategy*: Different from the existing results on event-based containment control problem of multi-agent systems [23]–[25], where the triggering mechanism is in the form of $u_i(t) = \sum_{j\in\mathcal{L}} a_{ij}[(\hat{x}_j(t) - h_j) - (\hat{x}_i(t) - h_i)], i \in \mathcal{L}$ and $u_i(t) = \sum_{j\in\mathcal{L}\bigcup\mathcal{F}} a_{ij}(\hat{x}_j(t) - \hat{x}_i(t)), i \in \mathcal{F}$, where $\hat{x}_i(t)$ is the latest broadcasting states of agent i, i.e., $\hat{x}_i(t) = x_i(t_k^i), t \in [t_k^i, t_{k+1}^i)$. In other words, the event-triggering mechanism in [23]–[25] can be rewritten as: when $t \in [t_k^i, t_{k+1}^i)$

$$u_i(t) = \sum_{j\in\mathcal{L}} a_{ij}[(x_j(t_{k''}^j) - h_j) - (x_i(t_k^i) - h_i)], i \in \mathcal{L},$$
$$u_i(t) = \sum_{j\in\mathcal{F}\bigcup\mathcal{L}} a_{ij}[(x_j(t_{k''}^j) - x_i(t_k^i)], i \in \mathcal{F}$$

where $k'' = \arg \min_{m\in\mathbb{N}, t_m^j \le t, j\in N_i} \{t - t_m^j\}$, while our event-triggering mechanism in (2) can be regarded as the following form ($\gamma = 0$): when $t \in [t_k^i, t_{k+1}^i)$

$$u_i(t) = \sum_{j\in\mathcal{L}} a_{ij}[(x_j(t_k^i) - h_j) - (x_i(t_k^i) - h_i)], i \in \mathcal{L},$$
$$u_i(t) = \sum_{j\in\mathcal{F}\bigcup\mathcal{L}} a_{ij}[(x_j(t_k^i) - x_i(t_k^i)], i \in \mathcal{F},$$

The triggering mechanism used in [23]–[25] can be regarded as a kind of push-based event-triggering mechanisms and the triggering mechanism in (2) is a kind of pull-based event-triggering mechanisms [26]. In the push-based event-triggering mechanism, each node uses the information of its local neighborhood at their latest triggered instants. In the pull-based event-triggering mechanism, each node uses the information of its local neighborhood only at the latest time of specific event of the node itself, i.e., the event-triggering controller in (2) will remain unchanged until its next triggering instant comes.

Denote by $\delta(t) = (\delta_{N-m+1}^T(t), \ldots, \delta_N^T(t))^T$ for convenience. Clearly, the Laplacian matrix of the multi-agent system in (2) can be partitioned into the following matrix:

$$L = \begin{bmatrix} L_1 & L_2 \\ 0_{m\times(N-m)} & L_3 \end{bmatrix}$$

where $L_1 \in \mathbb{R}^{(N-m)\times(N-m)}$, $L_2 \in \mathbb{R}^{(N-m)\times m}$, $L_3 \in \mathbb{R}^{m\times m}$. Denote by $\mathcal{G}_{\mathcal{L}}$ the interaction topology among the leaders. It is easy to see from the structure of L that L_3 is the Laplacian matrix corresponding to $\mathcal{G}_{\mathcal{L}}$. The main purpose of this paper is to present an effective pull-based event-triggering protocol such that the containment control problem of the multi-agent system in (1) can be solved. The following assumption, definitions and lemmas are needed to derive the main results of this paper.

Definition 1: Let $\mathcal{C}$ be a set in the real vector space $V \subseteq \mathbb{R}^n$. The set $\mathcal{C}$ is called convex if, for any x and y in $\mathcal{C}$, the point $(1 - z)x + zy$ is in $\mathcal{C}$ for any $z \in [0, 1]$. The convex hull for a set of points $X = \{x_1, \ldots, x_p\}$ in V is the minimal convex set containing all points in X. We use Co$\{X\}$ to denote the convex hull of X. In particular, $\mathrm{Co}\{X\} = \{\sum_{i=1}^p \alpha_i x_i | x_i \in X, \alpha_i \ge 0 \in \mathbb{R}, \sum_{i=1}^p \alpha_i = 1\}$.

Definition 2: The containment control problem of the multi-agent system in (1) with the event-triggering protocol in (2) is said to be solved, if each leader can track the desired state r_i and the followers can converge to the convex hull formed by the leaders.

Definition 3: For Case 2) ($\gamma = 0$): The containment control problem of the multi-agent system in (1) with the event-triggering protocol in (2) is said to be solved, if $\lim_{t\to+\infty} \|x_i(t) - r_i - x_j(t) + r_j\| \to 0$, $i, j = N - m + 1, \ldots, N$, and the followers can converge to the convex hull formed by the leaders.

Assumption 1: The directed graph $\mathcal{G}$ has a united spanning tree if $\mathcal{G}_{\mathcal{L}}$ has a directed spanning tree and for each follower, there exists at least one leader that has a directed path to the follower.

Lemma 1: [6] L_1 is invertible if and only if Assumption 1 is satisfied in the directed graph $\mathcal{G}$. Moreover, each element of $-L_1^{-1}L_2$ is nonnegative and each row of $-L_1^{-1}L_2$ has a sum equal to 1.

In the following, we will give some denotations, which are divided into three parts for easy derivations.

Part I: For each agent i, we define the measurement errors and their vectors as follows:

$$e_i(t) = x_i(t_k^i) - x_i(t), t \in [t_k^i, t_{k+1}^i), i \in \mathcal{F}$$
$$e_{ij}(t) = x_j(t_k^i) - x_j(t), t \in [t_k^i, t_{k+1}^i), i \in \mathcal{F}, j \in N_i$$
$$\tilde{e}_i(t) = (e_{i1}^T(t), \ldots, e_{i(i-1)}^T(t), e_{i(i+1)}^T(t), \ldots, e_{iN}^T(t))^T, i \in \mathcal{F}$$
$$\tilde{e}_{\mathcal{F}}(t) = (\tilde{e}_1^T(t), \ldots, \tilde{e}_{N-m}^T(t))^T$$
$$e_{\mathcal{F}}(t) = (e_1^T(t), \ldots, e_{N-m}^T(t))^T$$
$$\epsilon_i(t) = \delta_i(t_k^i) - \delta_i(t), i \in \mathcal{L}$$

$$\begin{aligned}\epsilon_{ij}(t) &= \delta_j(t_k^i) - \delta_j(t), i \in \mathcal{L}, j \in N_i\\ \tilde{\epsilon}_i(t) &= (\epsilon_{i(N-m+1)}^T(t), \ldots, \epsilon_{i(i-1)}^T(t)\\ &\qquad \epsilon_{i(i+1)}^T(t), \ldots, \epsilon_{iN}^T(t))^T, i \in \mathcal{L}\\ \tilde{\epsilon}_{\mathcal{L}}(t) &= (\tilde{\epsilon}_{N-m+1}^T(t), \ldots, \tilde{\epsilon}_N^T(t))^T\\ \epsilon_{\mathcal{L}}(t) &= (\epsilon_{N-m+1}^T(t), \ldots, \epsilon_N^T(t))^T\end{aligned}$$

where $e_i(t)$ and $e_{ij}(t)$ denote, respectively, the measurement errors of agent i and agent j in the followers' network; $\epsilon_i(t)$ and $\epsilon_{ij}(t)$ denote, respectively, the measurement errors of $\delta_i(t)$ and $\delta_j(t)$ in the leaders' network. The vectors $\tilde{e}_i(t), \tilde{e}_{\mathcal{F}}(t), e_{\mathcal{F}}(t), \tilde{\epsilon}_i(t), \tilde{\epsilon}_{\mathcal{L}}(t)$ and $\epsilon_{\mathcal{L}}(t)$ are presented here for easy derivations.

Part II: For $i \in \mathcal{F}$

$$\begin{aligned}a_i^{\mathcal{F}} &= (a_{i1}, \ldots, a_{i(i-1)}, a_{i(i+1)}, \ldots, a_{iN}), i \in \mathcal{F}\\ A_{\mathcal{F}} &= \mathrm{diag}\{a_1^{\mathcal{F}}, \ldots, a_{N-m}^{\mathcal{F}}\}\\ d_i^{\mathcal{F}} &= \sum_{j \in N_i, i \in \mathcal{F}} a_{ij}, D_{\mathcal{F}} = \mathrm{diag}\{d_1^{\mathcal{F}}, \ldots, d_{N-m}^{\mathcal{F}}\}\\ d_{\max}^{\mathcal{F}} &= \max_{i \in \mathcal{F}}\{d_i^{\mathcal{F}}\}, d_{\min}^{\mathcal{F}} = \min_{i \in \mathcal{F}}\{d_i^{\mathcal{F}}\}\\ b_i^{\mathcal{F}} &= \sqrt{\sum_{j \in N_i, i \in \mathcal{F}} a_{ij}^2}, a_{\max}^{\mathcal{F}} = \max_{i \in \mathcal{F}}\{b_i^{\mathcal{F}}\}\\ a_{\min}^{\mathcal{F}} &= \min_{i \in \mathcal{F}}\{b_i^{\mathcal{F}}\}\\ x_{\mathcal{F}}(t) &= (x_1^T(t), x_2^T(t), \ldots, x_{N-m}^T(t))^T\end{aligned}$$

where $d_{\max}^{\mathcal{F}}$ and $d_{\min}^{\mathcal{F}}$ denote, respectively, the maximum and minimum values of the in-degree of nodes in followers' network. $x_{\mathcal{F}}(t)$ is the vector form of the followers' states, and the other variables are given for easy derivations.

Part III: For $i \in \mathcal{L}$

$$\begin{aligned}a_i^{\mathcal{L}} &= (a_{i(N-m+1)}, \ldots, a_{i(i-1)}, a_{i(i+1)}, \ldots, a_{iN}), i \in \mathcal{L}\\ A_{\mathcal{L}} &= \mathrm{diag}\{a_{N-m+1}^{\mathcal{L}}, \ldots, a_N^{\mathcal{L}}\}\\ d_i^{\mathcal{L}} &= \sum_{j \in N_i, i \in \mathcal{L}} a_{ij}, D_{\mathcal{L}} = \mathrm{diag}\{d_{N-m+1}^{\mathcal{L}}, \ldots, d_N^{\mathcal{L}}\}\\ d_{\max}^{\mathcal{L}} &= \max_{i \in \mathcal{L}}\{d_i^{\mathcal{L}}\}, d_{\min}^{\mathcal{L}} = \min_{i \in \mathcal{L}}\{d_i^{\mathcal{L}}\}\\ b_i^{\mathcal{L}} &= \sqrt{\sum_{j \in N_i, i \in \mathcal{L}} a_{ij}^2}, a_{\max}^{\mathcal{L}} = \max_{i \in \mathcal{L}}\{b_i^{\mathcal{L}}\}\\ a_{\min}^{\mathcal{L}} &= \min_{i \in \mathcal{L}}\{b_i^{\mathcal{L}}\}\\ x_{\mathcal{L}}(t) &= (x_{N-m+1}^T(t), x_{N-m+2}^T(t), \ldots, x_N^T(t))^T\end{aligned}$$

where $d_{\max}^{\mathcal{L}}$ and $d_{\min}^{\mathcal{L}}$ denote, respectively, the maximum and minimum values of the in-degree of nodes in the leaders' network. According to the above denotations, the protocol in (2) can be rewritten as

$$\begin{aligned}u_i(t) &= \beta \sum_{j \in \mathcal{L}} a_{ij}(\delta_i(t) - \delta_j(t) + \epsilon_i(t) - \epsilon_{ij}(t))\\ &\quad + \gamma\, \delta_i(t) + \gamma\, \epsilon_i(t), i \in \mathcal{L},\\ u_i(t) &= -h \sum_{j=1}^{N} a_{ij}(x_i(t) - x_j(t) + e_i(t) - e_{ij}(t)), i \in \mathcal{F}.\end{aligned} \tag{4}$$

In view of (1) and (4), we have:

$$\begin{aligned}\dot{\delta}(t) = &-\beta\left(\left(L_3 + \frac{\gamma}{\beta} I_m\right) \otimes I_n\right)\delta(t) - \beta(D_{\mathcal{L}} \otimes I_n)\epsilon_{\mathcal{L}}(t)\\ &+ \beta(A_{\mathcal{L}} \otimes I_n)\tilde{\epsilon}_{\mathcal{L}}(t) - \gamma\, \epsilon_{\mathcal{L}}(t).\end{aligned} \tag{5}$$

It can be seen from Lemma 1 that under Assumption 1, each row of $-L_1^{-1}L_2$ has a sum equal to 1. Hence, in order to investigate that the followers will converge to the convex hull formed by the leaders, we denote the following denotations:

$$\varepsilon_{\mathcal{F}}(t) = x_{\mathcal{F}}(t) + (L_1^{-1}L_2 \otimes I_n)x_{\mathcal{L}}(t)$$

and therefore, the containment control problem of the multi-agent system in (1) with the protocol (2) can be transformed into the stability problems of (5) and the following system:

$$\begin{aligned}\dot{\varepsilon}_{\mathcal{F}}(t) = &-h(L_1 \otimes I_n)x_{\mathcal{F}}(t) - h(L_2 \otimes I_n)x_{\mathcal{L}}(t)\\ &- h(D_{\mathcal{F}} \otimes I_n)e_{\mathcal{F}}(t) + h(A_{\mathcal{F}} \otimes I_n)\tilde{e}_{\mathcal{F}}(t)\\ &+ (L_1^{-1}L_2 \otimes I_n)\dot{x}_{\mathcal{L}}(t)\\ = &-h(L_1 \otimes I_n)\varepsilon_{\mathcal{F}}(t) - h(D_{\mathcal{F}} \otimes I_n)e_{\mathcal{F}}(t)\\ &+ h(A_{\mathcal{F}} \otimes I_n)\tilde{e}_{\mathcal{F}}(t) + (L_1^{-1}L_2 \otimes I_n)\dot{x}_{\mathcal{L}}(t).\end{aligned} \tag{6}$$

From Assumption 1 and the definition of $\beta(L_3 + \frac{\gamma}{\beta})$, we know that all the eigenvalues of $-L_1$ and $-\beta(L_3 + \frac{\gamma}{\beta})$ are in the open left half plane. Then, according to the linear system theory [32], there exists $M > 1, \bar{M} > 1, \rho > 0$, and $\bar{\rho} > 0$ such that

$$\|e^{-h(L_1 \otimes I_n)(t-t_0)}\| \leq M e^{-\rho(t-t_0)}. \tag{7}$$

$$\|e^{\left(-\beta\left(L_3 + \frac{\gamma}{\beta}\right) \otimes I_n\right)(t-t_0)}\| \leq \bar{M} e^{-\bar{\rho}(t-t_0)}. \tag{8}$$

III. Main Results

A. Fixed Topology

In this section, the event-triggering containment control problem of multi-agent networks with fixed topology will be investigated. In the following, two different cases will be investigated, respectively: 1) Leaders are opinionated, i.e., $\gamma > 0$ is considered in (2) and (5); 2) Leaders are not opinionated, i.e., $\gamma = 0$ is considered in (2) and (5). For case 1), we will present the following theorem:

Theorem 1: Consider the multi-agent system in (1) with the protocol (2) with $\gamma > 0$. Suppose that Assumption 1 holds and event-triggering function is given as follows:

$$\begin{aligned}f_i &= d_i^{\mathcal{F}}\|e_i(t)\| + b_i^{\mathcal{F}}\|\tilde{e}_i(t)\| - \varrho_i c_2 e^{-\alpha(t-t_0)}\\ &\quad - (1 - \varrho_i)c_1 e^{-\alpha(t-t_0)}, i \in \mathcal{F},\\ f_i &= (d_i^{\mathcal{L}} + \gamma)\|\epsilon_i(t)\| + b_i^{\mathcal{L}}\|\tilde{\epsilon}_i(t)\| - \varrho_i c_2 e^{-\alpha(t-t_0)}\\ &\quad - (1 - \varrho_i)c_1 e^{-\alpha(t-t_0)}, i \in \mathcal{L}\end{aligned} \tag{9}$$

in which $0 < c_1 < c_2$, $0 < \varrho_i < 1, i = 1, 2, \ldots, N$, and $0 < \alpha < \min\{\rho, \bar{\rho}\}$. Then, each leader $x_i(t)$ will track the constant objective r_i, and the followers will converge to the convex hull formed by the leaders. Moreover, the final positions of the followers are given by $-(L_1^{-1}L_2 \otimes I_n)x_{\mathcal{L}}(t)$.

Proof: In the following, we will divide the proof into two steps. Firstly, we show that each leader $x_i(t)$ will track the constant objective r_i. Secondly, we aim to prove that all the followers will converge to the convex hull formed by the leaders.

Step I: In this step, we aim to show that each leader $x_i(t)$ will track the constant objective r_i. It can be obtained from (5) that

$$\delta(t) = e^{-\zeta(t-t_0)}\delta(t_0) - \beta\int_{t_0}^{t} e^{-\zeta(t-s)}[(D_{\mathcal{L}}\otimes I_n)\epsilon_{\mathcal{L}}(s) - (A_{\mathcal{L}}\otimes I_n)\tilde{\epsilon}_{\mathcal{L}}(s)]ds - \gamma\int_{t_0}^{t} e^{-\zeta(t-s)}\tilde{\epsilon}_{\mathcal{L}}(s)ds \tag{10}$$

where $\zeta = \beta(L_3 + \frac{\gamma}{\beta}I_m)\otimes I_n$. In view of (8) and (10), we have

$$\begin{aligned}\|\delta(t)\| \le& \bar{M}e^{-\bar{\rho}(t-t_0)}\|\delta(t_0)\| + \beta\bar{M}\int_{t_0}^{t} e^{-\bar{\rho}(t-s)}[\|D_{\mathcal{L}}\| \\ &\times \|\epsilon_{\mathcal{L}}(s)\| + \|A_{\mathcal{L}}\|\|\tilde{\epsilon}_{\mathcal{L}}(s)\|]ds \\ &+ \gamma\bar{M}\int_{t_0}^{t} e^{-\bar{\rho}(t-s)}\|\epsilon_{\mathcal{L}}(s)\|ds. \end{aligned}\tag{11}$$

In the following, the purpose of the subsequent manipulations is to find the upper bounds of each term in (11), which would allow to conclude that $\delta(t)$ vanishes as $t\to\infty$ and therefore we can show that each leader $x_i(t)$ will track the constant objective r_i.

For $t\in[t_k^i, t_{k+1}^i)$, $i\in\mathcal{L}$, the triggering function in (9) enforces that

$$\begin{aligned}\gamma\|\epsilon_i(t)\| \le& (1-\varrho_i)c_1e^{-\alpha(t-t_0)} + \varrho_i c_2 e^{-\alpha(t-t_0)} \\ <& (1-\varrho_i)c_2e^{-\alpha(t-t_0)} + \varrho_i c_2 e^{-\alpha(t-t_0)} \\ =& c_2e^{-\alpha(t-t_0)}.\end{aligned}\tag{12}$$

It can be obtained from (12) that

$$\gamma\|\epsilon_{\mathcal{L}}(t)\| \le \sqrt{m}c_2e^{-\alpha(t-t_0)}. \tag{13}$$

In addition, for $t\in[t_k^i, t_{k+1}^i), i\in\mathcal{L}$, the triggering function enforces

$$d_i^{\mathcal{L}}\|\epsilon_i(t)\| \le c_2e^{-\alpha(t-t_0)}. \tag{14}$$

Hence,

$$\sum_{i=N-m+1}^{N}(d_i^{\mathcal{L}})^2\|\epsilon_i(t)\|^2 \le mc_2^2e^{-2\alpha(t-t_0)}. \tag{15}$$

From (15), it yields

$$d_{\max}^{\mathcal{L}}\|\epsilon_{\mathcal{L}}(t)\| \le \frac{d_{\max}^{\mathcal{L}}}{d_{\min}^{\mathcal{L}}}c_2\sqrt{m}e^{-\alpha(t-t_0)}. \tag{16}$$

Similarly

$$a_{\max}^{\mathcal{L}}\|\tilde{\epsilon}_{\mathcal{L}}(t)\| \le \frac{a_{\max}^{\mathcal{L}}}{a_{\min}^{\mathcal{L}}}c_2\sqrt{m}e^{-\alpha(t-t_0)}. \tag{17}$$

It can be obtained from (16) and (17) that

$$\begin{aligned}&\|D_{\mathcal{L}}\otimes I_n\|\|\epsilon_{\mathcal{L}}(s)\| + \|A_{\mathcal{L}}\otimes I_n\|\|\tilde{\epsilon}_{\mathcal{L}}(s)\| \\ &= d_{\max}^{\mathcal{L}}\|\epsilon_{\mathcal{L}}(s)\| + a_{\max}^{\mathcal{L}}\|\tilde{\epsilon}_{\mathcal{L}}(s)\| \\ &\le \bar{\kappa}c_2\sqrt{m}e^{-\alpha(t-t_0)}\end{aligned}\tag{18}$$

where $\bar{\kappa} = 2\max\{\frac{a_{\max}^{\mathcal{L}}}{a_{\min}^{\mathcal{L}}}, \frac{d_{\max}^{\mathcal{L}}}{d_{\min}^{\mathcal{L}}}\}$.

In view of (11), (13), and (18), we get

$$\begin{aligned}\|\delta(t)\| \le& \bar{M}e^{-\bar{\rho}(t-t_0)}\|\delta(t_0)\| \\ &+ c_2\beta\bar{M}\bar{\kappa}\sqrt{m}\int_{t_0}^{t} e^{-\bar{\rho}(t-s)}e^{-\alpha(s-t_0)}ds \\ &+ c_2\sqrt{m}\bar{M}\int_{t_0}^{t} e^{-\bar{\rho}(t-s)}e^{-\alpha(s-t_0)}ds.\end{aligned}\tag{19}$$

From (19), we get

$$\begin{aligned}\|\delta(t)\| \le& e^{-\bar{\rho}(t-t_0)}\left(\bar{M}\|\delta(t_0)\| - \frac{c_2\beta\bar{M}\bar{\kappa}\sqrt{m} + c_2\sqrt{m}\bar{M}}{\bar{\rho}-\alpha}\right) \\ &+ c_2\beta\sqrt{m}\bar{M}\bar{\kappa}\frac{e^{-\alpha(t-t_0)}}{\bar{\rho}-\alpha} + c_2\sqrt{m}\bar{M}\frac{e^{-\alpha(t-t_0)}}{\bar{\rho}-\alpha}.\end{aligned}\tag{20}$$

It is easy to see from (20) that as $t\to\infty$, $\delta(t)\to 0$. Hence, each leader $x_i(t)$ can track the constant objective r_i.

Step II: In this step, we will prove that the followers will converge to the convex hull formed by the leaders.

From systems (5) and (6), one gets

$$\begin{aligned}\varepsilon_{\mathcal{F}}(t) =& e^{-h(L_1\otimes I_n)(t-t_0)}\varepsilon_{\mathcal{F}}(t_0) - h\int_{t_0}^{t} e^{-h(L_1\otimes I_n)(t-s)} \\ &\times\big[(D_{\mathcal{F}}\otimes I_n)e_{\mathcal{F}}(s) - (A_{\mathcal{F}}\otimes I_n)\tilde{e}_{\mathcal{F}}(s)]ds \\ &+ \int_{t_0}^{t} e^{-h(L_1\otimes I_n)(t-s)}(L_1^{-1}L_2\otimes I_n)\dot{x}_{\mathcal{L}}(s)ds \\ =& e^{-h(L_1\otimes I_n)(t-t_0)}\varepsilon_{\mathcal{F}}(t_0) - h\int_{t_0}^{t} e^{-h(L_1\otimes I_n)(t-s)} \\ &\times\big[(D_{\mathcal{F}}\otimes I_n)e_{\mathcal{F}}(s) - (A_{\mathcal{F}}\otimes I_n)\tilde{e}_{\mathcal{F}}(s)\big]ds \\ &+ \beta\int_{t_0}^{t} e^{-h(L_1\otimes I_n)(t-s)}(L_1^{-1}L_2\otimes I_n)\big[\zeta\delta(s) \\ &+ (D_{\mathcal{L}}\otimes I_n)\epsilon_{\mathcal{L}}(s) - (A_{\mathcal{L}}\otimes I_n)\tilde{\epsilon}_{\mathcal{L}}(s))\big]ds \\ &+ \gamma\int_{t_0}^{t} e^{-h(L_1\otimes I_n)(t-s)}(L_1^{-1}L_2\otimes I_n)\epsilon_{\mathcal{L}}(s)\big]ds\end{aligned}\tag{21}$$

where the second equality is satisfied since $\dot{x}_{\mathcal{L}}(t) = -\dot{\delta}(t)$. In view of (7) and (21), we have

$$\begin{aligned}\|\varepsilon_{\mathcal{F}}(t)\| \le& e^{-\rho(t-t_0)}M\|\varepsilon_{\mathcal{F}}(t_0)\| + hM\int_{t_0}^{t} e^{-\rho(t-s)}\big[\|D_{\mathcal{F}}\| \\ &\times\|e_{\mathcal{F}}(s)\| + \|A_{\mathcal{F}}\|\|\tilde{e}_{\mathcal{F}}(s)\|\big]ds \\ &+ \beta\|L_1^{-1}L_2\|M\int_{t_0}^{t} e^{-\rho(t-s)}\big[\|\zeta\delta(s)\| \\ &+ \|D_{\mathcal{L}}\|\|\epsilon_{\mathcal{L}}(s)\| + \|A_{\mathcal{L}}\|\|\tilde{\epsilon}_{\mathcal{L}}(s)\|\big]ds \\ &+ \gamma M\|L_1^{-1}L_2\|\int_{t_0}^{t} e^{-\rho(t-s)}\|\epsilon_{\mathcal{L}}(s)\|\big]ds.\end{aligned}\tag{22}$$

Similar to the proof of (18), we have for $t\in[t_k^i, t_{k+1}^i)$, $i\in\mathcal{F}$

$$\begin{aligned}&\|D_{\mathcal{F}}\|\|\epsilon_{\mathcal{F}}(s)\| + \|A_{\mathcal{F}}\|\|\tilde{\epsilon}_{\mathcal{F}}(s)\| \\ &= d_{\max}^{\mathcal{F}}\|\epsilon_{\mathcal{L}}(s)\| + a_{\max}^{\mathcal{F}}\|\tilde{\epsilon}_{\mathcal{F}}(s)\| \\ &\le \kappa c_2\sqrt{N-m}e^{-\alpha(t-t_0)}\end{aligned}\tag{23}$$

where $\kappa = 2\max\{\frac{a_{\max}^{\mathcal{F}}}{a_{\min}^{\mathcal{F}}}, \frac{d_{\max}^{\mathcal{F}}}{d_{\min}^{\mathcal{F}}}\}$.

In addition, from (20) and noting that $e^{-\bar{\rho}(t-t_0)} \leq e^{-\alpha(t-t_0)}$, we can obtain

$$\begin{aligned}\|\delta(s)\| \leq & \bar{M}e^{-\alpha(s-t_0)}\|\delta(t_0)\| + c_2\beta\sqrt{m}\bar{M}\bar{\kappa}\frac{e^{-\alpha(s-t_0)}}{\bar{\rho}-\alpha} \\ & + c_2\sqrt{m}\bar{M}\frac{e^{-\alpha(s-t_0)}}{\bar{\rho}-\alpha} \\ = & \Upsilon e^{-\alpha(s-t_0)} \end{aligned} \tag{24}$$

where $\Upsilon = \bar{M}\|\delta(t_0)\| + \frac{c_2\beta\sqrt{m}\bar{M}\bar{\kappa}+c_2\sqrt{m}\bar{M}}{\bar{\rho}-\alpha}$. From (23) and (24), it follows from (22)

$$\begin{aligned}\|\varepsilon_{\mathcal{F}}(t)\| \leq & Me^{-\rho(t-t_0)}\|\varepsilon_{\mathcal{F}}(t_0)\| + hM\int_{t_0}^{t} e^{-\rho(t-s)}\Big[d_{\max}^{\mathcal{F}} \\ & \times \|e_{\mathcal{F}}(s)\| + a_{\max}^{\mathcal{F}}\|\tilde{e}_{\mathcal{F}}(s)\|\Big]ds + \beta M\|L_1^{-1}L_2\| \\ & \times \int_{t_0}^{t} e^{-\rho(t-s)}e^{-\alpha(s-t_0)}\big[\|\zeta\|\Upsilon + \bar{\kappa}\sqrt{m}c_2\big]ds \\ & + M\|L_1^{-1}L_2\|\int_{t_0}^{t} e^{-\rho(t-s)}e^{-\alpha(s-t_0)}\sqrt{m}c_2)ds. \end{aligned} \tag{25}$$

From (22) –(25), one has

$$\begin{aligned}\|\varepsilon_{\mathcal{F}}(t)\| \leq & Me^{-\rho(t-t_0)}\|\varepsilon_{\mathcal{F}}(t_0)\| \\ & + c_2hM\kappa\sqrt{N-m}\int_{t_0}^{t} e^{-\rho(t-s)}e^{-\alpha(s-t_0)}ds \\ & + \beta M\|L_1^{-1}L_2\|\left(\|\zeta\|\Upsilon + \bar{\kappa}\sqrt{m}c_2 + \frac{\sqrt{m}c_2}{\beta}\right) \\ & \times \int_{t_0}^{t} e^{-\rho(t-s)}e^{-\alpha(s-t_0)}ds. \end{aligned} \tag{26}$$

By means of (26), one gets

$$\begin{aligned}\|\varepsilon_{\mathcal{F}}(t)\| \leq & e^{-\rho(t-t_0)}\Bigg(M\|\varepsilon_{\mathcal{F}}(t_0)\| - hM\kappa\sqrt{N-m}\frac{c_2}{\rho-\alpha} \\ & - \frac{\beta M\|L_1^{-1}L_2\|\left(\|\zeta\|\Upsilon + \sqrt{m}c_2\bar{\kappa} + \frac{\sqrt{m}c_2}{\beta}\right)}{\rho-\alpha}\Bigg) \\ & + e^{-\alpha(t-t_0)}\Bigg(\frac{c_2hM\kappa\sqrt{N-m}}{\rho-\alpha} \\ & + \frac{\beta M\|L_1^{-1}L_2\|\left(\|\zeta\|\Upsilon + \bar{\kappa}\sqrt{m}c_2 + \frac{\sqrt{m}c_2}{\beta}\right)}{\rho-\alpha}\Bigg). \end{aligned} \tag{27}$$

From (27), we know that as $t \to \infty$, $\varepsilon_{\mathcal{F}}(t) \to 0$ and therefore the final positions of the followers converge to $-(L_1^{-1}L_2 \otimes I_n)x_{\mathcal{L}}(t)$. In addition, from Lemma 1, we know that each element of the matrix $-L_1^{-1}L_2$ is nonnegative and each row of $-L_1^{-1}L_2$ has a sum equal to 1. Thus, from Definition 1, we can conclude that all the followers converge to the convex hull formed by the leaders. ■

Theorem 2: Under the assumptions in Theorem 1, the event-triggering algorithm in (9) with the updating rule (3) will not exhibit Zeno behaviors.

Proof: We first show that Zeno behaviors can be avoided in the leaders' network and then by repeating similar steps, we can prove that Zeno behaviors can be avoided in the followers' network.

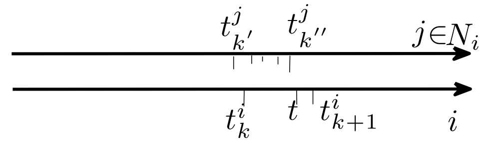


Fig. 1. Relationship between k' and k''.

For $i \in \mathcal{L}$ and $t \in [t_k^i, t_{k+1}^i)$, we have $\dot{\epsilon}_i(t) = -\dot{\delta}_i(t) = u_i(t_k^i)$. By considering $\epsilon_i(t_k^i) = 0$, we get

$$\|\epsilon_i(t)\| \leq \int_{t_k^i}^{t} \|u_i(t_k^i)\|ds. \tag{28}$$

From (5), (13), (18), and (24), we get

$$\begin{aligned}\|u_i(t_k^i)\| \leq & \|\dot{\delta}(t)\| \\ \leq & \beta\|L_3 + \frac{\gamma}{\beta}I_m\|\|\delta(t)\| + \beta\|D_{\mathcal{L}}\|\|\epsilon_{\mathcal{L}}(t)\| \\ & + \beta\|A_{\mathcal{L}}\|\|\tilde{\epsilon}_{\mathcal{L}}(t)\| + \gamma\,\|\epsilon_{\mathcal{L}}(t)\| \\ \leq & \bar{M}\|\zeta\|\|\delta(t_0)\|e^{-\alpha(t-t_0)} + c_2\beta\sqrt{m}\bar{M}\bar{\kappa}\frac{e^{-\alpha(t-t_0)}}{\bar{\rho}-\alpha} \\ & + c_2\sqrt{m}\bar{M}\frac{e^{-\alpha(t-t_0)}}{\bar{\rho}-\alpha} + c_2\beta\sqrt{m}\bar{\kappa}e^{-\alpha(t-t_0)} \\ & + c_2\sqrt{m}e^{-\alpha(t-t_0)}. \end{aligned} \tag{29}$$

Denote

$$\begin{aligned}\gamma_1 = & \frac{c_2\beta\sqrt{m}\bar{M}\bar{\kappa}}{\bar{\rho}-\alpha} + c_2\sqrt{m} \\ & + \frac{c_2\sqrt{m}\bar{M}}{\bar{\rho}-\alpha} + c_2\beta\sqrt{m}\bar{\kappa}\end{aligned}$$

and noting that $e^{-\alpha(t-t_0)} \leq e^{-\alpha(t_k^i-t_0)}, t \in [t_k^i, t_{k+1}^i)$, then, the following inequality holds from (28) and (29):

$$\begin{aligned}\|\epsilon_i(t)\| \leq & \|u_i(t_k^i)\|(t-t_k^i) \\ \leq & (t-t_k^i)\big[\gamma_1e^{-\alpha(t_k^i-t_0)} + \bar{M}\|\zeta\|\|\delta(t_0)\|e^{-\alpha(t_k^i-t_0)}\big] \\ \leq & 2(t-t_k^i)\gamma_2e^{-\alpha(t_k^i-t_0)} \end{aligned} \tag{30}$$

where $\gamma_2 = \max\{\gamma_1, \bar{M}\|\zeta\|\|\delta(t_0)\|\}$.

For $i \in \mathcal{L}$, let $k' = \arg\min_{p\in\mathbb{N}, t_k^i\geq t_p^j, j\in N_i}\{t_k^i - t_p^j\}$, $k'' = \arg\min_{m\in\mathbb{N}, t_m^j\leq t, j\in N_i}\{t - t_m^j\}$. One can better understand k' and k'' from Fig. 1

$$\dot{\epsilon}_{ij}(t) = \begin{cases} u_j(t_{k''}^j), & t \geq t_{k''}^j, \\ u_j(t_{k''-1}^j), & t \in [t_{k''-1}^j, t_{k''}^j), \\ \ldots, \\ u_j(t_{k'}^j), & t \in [t_k^i, t_{k'+1}^j). \end{cases} \tag{31}$$

Similar to (29), for $k' \leq p \leq k''$, $t \in [t_p^j, t_{p+1}^j)\bigcap[t_k^i, t_{k+1}^i)$, we have

$$\begin{aligned}\|u_j(t_p^j)\| \leq & 2\gamma_2e^{-\alpha(t-t_0)} \\ \leq & 2\gamma_2e^{-\alpha(t_k^i-t_0)}. \end{aligned} \tag{32}$$

Hence, for $t \in [t_k^i, t_{k+1}^i)$, it can be seen from (32) and Fig. 1

$$\begin{aligned}\|\epsilon_{ij}(t)\| = &\|u_j(t_{k''}^j)(t - t_{k''}^j) \\ &+ u_j(t_{k''-1}^j)(t_{k''}^j - t_{k''-1}^j) \\ &+ \dots \\ &+ u_j(t_{k'}^j)(t_{k'+1}^j - t_k^i)\| \\ \le &2\gamma_2(t - t_k^i)e^{-\alpha(t_k^i - t_0)}. \end{aligned} \tag{33}$$

From (33), we obtain

$$\|\tilde{\epsilon}_i(t)\| \le 2\sqrt{|N_j|}\gamma_2(t - t_k^i)e^{-\alpha(t_k^i - t_0)}. \tag{34}$$

In addition, from (30) and (33)

$$\begin{aligned}&d_i^{\mathcal{L}}\|\epsilon_i(t)\| + b_i^{\mathcal{L}}\|\tilde{\epsilon}_i(t)\| \\ &\le\big[2d_i^{\mathcal{L}}\gamma_2 + 2b_i^{\mathcal{L}}\sqrt{|N_j|}\gamma_2\big](t - t_k^i)e^{-\alpha(t_k^i - t_0)}. \end{aligned} \tag{35}$$

Note that $c_1e^{-\alpha(t-t_0)} \le (1-\varrho_i)c_1e^{-\alpha(t-t_0)} + \varrho_i c_2 e^{-\alpha(t-t_0)}$. Hence, the next event will not be triggered before $d_i^{\mathcal{L}}\|\epsilon_i(t)\| + b_i^{\mathcal{L}}\|\tilde{\epsilon}_i(t)\| = c_1e^{-\alpha(t-t_0)}$ is satisfied. Hence, a lower bound on the event intervals is given by $\tau = t - t_k^i$ that solves the equation

$$c_1e^{-\alpha\tau} = \big[2d_i^{\mathcal{L}}\gamma_2 + 2b_i^{\mathcal{L}}\sqrt{|N_j|}\gamma_2\big]\tau. \tag{36}$$

We are now in a position to prove that the solution of (36) is strictly positive. Let

$$g(\tau) = c_1e^{-\alpha\tau} - \big[2d_i^{\mathcal{L}}\gamma_2 + 2b_i^{\mathcal{L}}\sqrt{|N_j|}\gamma_2\big]\tau.$$

Then $\dot{g}(\tau) = -\alpha c_1 e^{-\alpha\tau} - 2d_i^{\mathcal{L}}\gamma_2 - 2b_i^{\mathcal{L}}\sqrt{|N_j|}\gamma_2 < 0$, which means that $g(\tau)$ is a strictly decreasing function. Since $g(0) = c_1 > 0$, we obtain that the solution τ of $g(\tau)$ is strictly positive, which means that Zeno behaviors in the leaders' network can be avoided. Similarly, we can show that Zeno behaviors can be avoided in the followers' network. This completes the proof. ■

Remark 2: In this paper, a pull-based event-triggering protocol is developed to solve the containment control problem of multi-agent networks, in which each agent updates its state when an exponentially decreasing threshold is reached. The advantage of the triggering function is to enforce the sum of each agent's state error and it's neighbors' errors to be less than the prescribed threshold $\varrho_i c_2 e^{-\alpha(t-t_0)} + (1-\varrho_i)c_1e^{-\alpha(t-t_0)}$, which plays an important role in proving the exponential convergence. Different from the triggering function proposed in [20], where each agent share the same triggering function, in this paper, each agent does not share the same triggering function, which extends the triggering function in [20] into a more general case.

Remark 3: In some cases, the network topology may be nonlinear [33], [34]. In addition, in practice, noises or disturbances widely exist in many practical systems [35]–[37], and therefore it is interesting to investigate event-triggering containment control of multi-agent systems under noises or different topologies. However, as the main purpose of this paper is to investigate the event-triggering control of multi-agent systems, we do not consider noises or disturbances here and we will consider them by using the methods in [35]–[37], [33]–[34] in the near future.

Theorem 3: Consider the multi-agent system in (1) with the protocol (2) with $\gamma = 0$. Suppose that Assumption 1 holds and the triggering function is given as follows:

$$\begin{aligned}f_i = &d_i^{\mathcal{F}}\|e_i(t)\| + b_i^{\mathcal{F}}\|\tilde{e}_i(t)\| - \varrho_i c_2 e^{-\alpha(t-t_0)} \\ &- (1-\varrho_i)c_1e^{-\alpha(t-t_0)}, i \in \mathcal{F}, \\ f_i = &d_i^{\mathcal{L}}\|\epsilon_i(t)\| + b_i^{\mathcal{L}}\|\tilde{\epsilon}_i(t)\| - \varrho_i c_2 e^{-\alpha(t-t_0)} \\ &- (1-\varrho_i)c_1e^{-\alpha(t-t_0)}, i \in \mathcal{L}.\end{aligned}$$

Then, under the updating rule (3), we have $\lim_{t\to+\infty}\|x_i(t) - r_i - x_j(t) + r_j\| \to 0$, $(i, j = N - m + 1, \dots, N)$, and the followers will converge to the convex hull formed by the leaders with the final positions $-(L_1^{-1}L_2 \otimes I_n)x_{\mathcal{L}}(t)$. In addition, Zeno behaviors can be avoided.

Proof: The proof is similar to the proof of Theorem 1 and is omitted here. ■

B. Switching Topology

In Theorems 1 and 3, the event-triggering containment control problem is investigated for multi-agent systems with fixed topology. However, in practice, due to the finite interaction region of sensing devices, link failures or creations in an interaction network might exist [38], [39]. Hence, it is of great importance to investigate containment control of multi-agent systems with switching topology. The switching topology of multi-agent systems is defined as follows: Let $\hat{\mathcal{G}} = \{\mathcal{G}^1, \dots, \mathcal{G}^l\}$ denote the set of all possible directed interaction graphs for the network of all the agents. In this paper, without loss of generality, we assume that interaction graphs and weighting factors change only at discrete times t_k, where $\{t_k\}$ is an infinite sequence satisfying $t_0 < t_1 < t_2 < \dots$. Denote by $\mathcal{G}(t)$ the network topology of the switched multi-agent system at time t and $\mathcal{A}(t) = [a_{ij}(t)]_{N\times N}$ is the weighted adjacency matrix of the switched multi-agent system at time t, where $a_{ij}(t)$ is the coupling coefficient from agent j to i at time t. Let $a_{ii}(t) = -\sum_{j=1}^N a_{ij}(t)$. In order to investigate the event-triggering containment control of the multi-agent system with switching topology, we need the following definitions and lemmas:

Definition 4: For some constant $\delta > 0$, the time-varying $\mathcal{G}(t)$ is said to have a δ-spanning tree across the time interval $[T_0, T_1)$, if $[\int_{T_0}^{T_1}\mathcal{A}(s)ds]^\delta$ has a spanning tree, where $A(t)^\delta = [a_{ij}(t)^\delta]$ is defined as

$$a_{ij}(t)^\delta = \begin{cases}0, & a_{ij}(t) < \delta \\ a_{ij}(t), & a_{ij}(t) \ge \delta.\end{cases}$$

Definition 5: For a given matrix $A = \big[A_1^T\ A_2^T\ \dots\ A_n^T\big]^T$, the Hajnal diameter of A is defined as $\mathrm{diam}(A) = \max_{i,j}\|A_i - A_j\|_\infty$.

Now, we will introduce the definition of Hajnal's inequality. An $N\times N$ matrix B is called as a stochastic matrix if the matrix B has a row sum of 1 and each element of B is not less than 0. The scrambling coefficient of a stochastic matrix $B = [B_{ij}]$ is defined as

$$\lambda(B) = \min_{i,l}\sum_j \min\{B_{ij}, B_{lj}\}.$$

Moreover, if $\lambda(B) > \delta$, then we say $\lambda(B)$ is δ-scrambling. Then, we get that the following Hajnal's inequality holds [26]

$$\mathrm{diam}\{AB\} \leq (1 - \lambda(A))\mathrm{diam}(B).$$

Lemma 2 ([26]) Let $\Phi(t, t_0)$ be the solution matrix of system $\dot{x}(t) = \dot{\mathcal{A}}(t)x(t)$ initiated at t_0. If the graph $\mathcal{G}(t)$ has a δ-spanning tree across time interval $[T_1, T_2)$, then $\Phi(T_2, T_1)$ is a stochastic matrix and it has a δ'-spanning tree with $\delta' = \min\{1, \delta\}e^{-(N-1)M_1(T_2-T_1)}$, where M_1 is the upper bound of $|a_{ij}(t)|$, i.e., $|a_{ij}(t)| < M_1, i, j = 1, 2, \ldots, N$. Moreover, the diagonal elements $\Phi_{i,i}(T_2, T_1) \geq \delta'$ holds for all $i = 1, \ldots, N$.

Assumption 2: If there exist a positive constant T and time sequence $\{T_k\}$ with $T_{k+1} - T_k < T, k = 0, 1, \ldots$, such that the leaders' graph has a δ-spanning tree across $[T_k, T_{k+1})$, and at any time t, for each follower, there exists at least one leader that has a directed path to the follower.

Then by repeating the similar steps as in (5) and (6), the following two models can be obtained:

$$\begin{aligned}\dot{\delta}(t) =& -\beta\left(\left(L_3(t) + \frac{\gamma}{\beta}I_m\right) \otimes I_n\right)\delta(t) + \beta(A_{\mathcal{L}}(t) \otimes I_n)\\ &\times \tilde{\epsilon}_{\mathcal{L}}(t) - \beta(D_{\mathcal{L}}(t) \otimes I_n)\epsilon_{\mathcal{L}}(t) - \gamma\epsilon_{\mathcal{L}}(t), \quad (37)\\ \dot{\hat{\varepsilon}}_{\mathcal{F}}(t) =& -h(L_1(t) \otimes I_n)\hat{\varepsilon}_{\mathcal{F}}(t) - h(D_{\mathcal{F}}(t) \otimes I_n)e_{\mathcal{F}}(t)\\ &+ h(A_{\mathcal{F}}(t) \otimes I_n)\tilde{e}_{\mathcal{F}}(t) + (L_1^{-1}(t)L_2(t) \otimes I_n)\dot{x}_{\mathcal{L}}(t), \quad (38)\end{aligned}$$

where $\hat{\varepsilon}(t) = x_{\mathcal{F}}(t) + (L_1^{-1}(t)L_2(t) \otimes I_n)x_{\mathcal{L}}(t)$. For the case of $\gamma > 0$, it is easy to get that the containment control can be obtained by using the switching system approach and repeating the similar steps in Theorem 1. Thus, in what follows, we mainly focus on the case of $\gamma = 0$.

For $\gamma = 0$, the following theorem can be obtained for containment control of the multi-agent system in (1) with switching topology.

Theorem 4: Consider the multi-agent system in (1) with switching topology with $\gamma = 0$. Suppose that Assumption 2 holds and $t_{k+1} - t_k > \frac{\ln \mathcal{M}}{\lambda}$ and the event-triggering function is given as follows:

$$\begin{aligned}f_i =& d_i^{\mathcal{F}}(t)\|e_i(t)\| + b_i^{\mathcal{F}}(t)\|\tilde{e}_i(t)\| - \hat{\varrho}_i\hat{c}_2e^{-\hat{\alpha}(t-t_0)}\\ &- (1 - \hat{\varrho}_i)\hat{c}_1e^{-\hat{\alpha}(t-t_0)}, i \in \mathcal{F},\\ f_i =& (d_i^{\mathcal{L}}(t))\|\epsilon_i(t)\| + b_i^{\mathcal{L}}(t)\|\tilde{\epsilon}_i(t)\| - \hat{\varrho}_i\hat{c}_2e^{-\hat{\alpha}(t-t_0)}\\ &- (1 - \hat{\varrho}_i)\hat{c}_1e^{-\hat{\alpha}(t-t_0)}, i \in \mathcal{L} \quad (39)\end{aligned}$$

where $0 < \hat{c}_1 < \hat{c}_2$, $0 < \hat{\varrho}_i < 1, i = 1, 2, \ldots, N$, $0 < \hat{\alpha} < c$, and $c = -\ln((1 - (\delta')^{N-m})^{\frac{1}{(N-m)T}})$. $\mathcal{M} = \max_{p=1,2,\ldots,l}\{M_p\}$, $\lambda = \min_{p=1,2,\ldots,l}\{\lambda_p\}$. λ_p, and M_p are defined as $e^{-h(L_1(t)\otimes I_n)(t-t_k)} \leq M_pe^{-\lambda_p(t-t_k)}, t \in [t_k, t_{k+1})$. Then, under the updating rule (3), we have $\lim_{t\to+\infty} \|x_i(t) - r_i - x_j(t) + r_j\| \to 0$, $(i, j = N - m + 1, \ldots, N)$, and the followers will converge to the convex hull formed by the leaders with the final positions $-(L_1^{-1}(t)L_2(t) \otimes I_n)x_{\mathcal{L}}(t)$. In addition, the Zeno behavior can be avoided.

Proof: Let $\Phi_{\mathcal{L}}(t, t_0)$ be the solution matrix of $\dot{\delta}(t) = -\beta(L_3(t) \otimes I_n)\delta(t)$. Then, we have

$$\begin{aligned}\delta(t) = \Phi_{\mathcal{L}}(t, t_0)\delta(t_0) - \beta\int_{t_0}^{t} \Phi_{\mathcal{L}}(t, s)[(D_{\mathcal{L}}(s) \otimes I_n)\epsilon_{\mathcal{L}}(s)\\ - (A_{\mathcal{L}}(s) \otimes I_n)\tilde{\epsilon}_{\mathcal{L}}(s)]ds. \quad (40)\end{aligned}$$

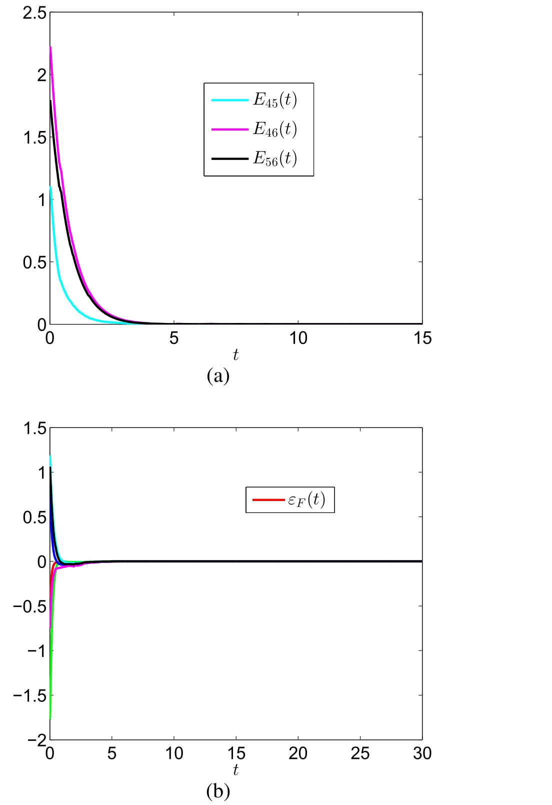


Fig. 2. (a) State trajectories for E_{45}, E_{46} and E_{56} with fixed topology; (b) State trajectories for $\varepsilon_{\mathcal{F}}(t)$ with fixed topology.

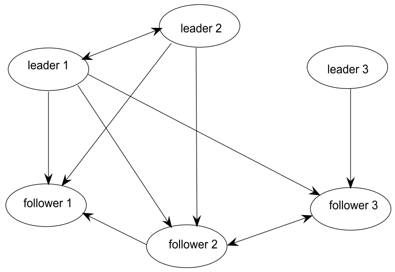
Due to the fact that for any $\eta, \zeta \in \mathbb{R}^n$, $\mathrm{diam}(\eta + \zeta) \leq \mathrm{diam}(\eta) + \mathrm{diam}(\zeta)$, we can obtain

$$\begin{aligned}\mathrm{diam}(\delta(t)) \leq& \mathrm{diam}(\Phi_{\mathcal{L}}(t, t_0)\delta(t_0)) + \mathrm{diam}\Big\{\beta\int_{t_0}^{t} \Phi_{\mathcal{L}}(t, s)\\ &\times [(D_{\mathcal{L}}(s) \otimes I_n)\epsilon_{\mathcal{L}}(s)\\ &- (A_{\mathcal{L}}(s) \otimes I_n)\tilde{\epsilon}_{\mathcal{L}}(s)]ds\Big\}. \quad (41)\end{aligned}$$

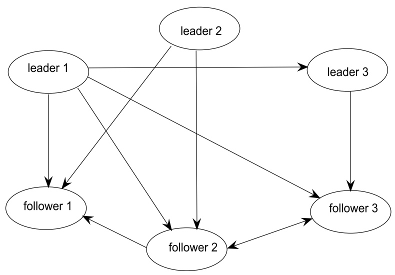
According to [26], we can get that the product of $\Phi(T_{k+1}, T_k)$ with length $N - m$ is $(\delta')^{N-m}$-scrambling. For $t \in [T_k, T_{k+1})$ with $k = \lfloor\frac{k}{N-m}\rfloor + d$, one has

$$\begin{aligned}\mathrm{diam}(\Phi_{\mathcal{L}}(t, t_0)) =& \mathrm{diam}(\Phi_{\mathcal{L}}(t, T_k)\Phi_{\mathcal{L}}(T_k, T_{k-1})\ldots\\ &\times \Phi_{\mathcal{L}}(T_1, T_0)\Phi_{\mathcal{L}}(T_0, t_0))\\ \leq& (1 - (\delta')^{N-m})^{\lfloor\frac{t-t_0}{(N-m)T}\rceil}\\ \leq& \frac{1}{1 - (\delta')^{N-m}}\left((1 - (\delta')^{N-m})^{\frac{1}{(N-m)T}}\right)^{t-t_0}\\ \leq& \frac{1}{1 - (\delta')^{N-m}}e^{-c(t-t_0)}. \quad (42)\end{aligned}$$

Then by following the similar steps in [26] and Theorem 1, we can get that $\lim_{t\to+\infty} \|x_i(t) - r_i - x_j(t) + r_j\| \to 0$, $(i, j = N - m + 1, \ldots, N)$. As the interaction topologies only change at the discrete-time times t_k and at any time t, for each follower, there exist at least one leader that has a directed path to follower. Thus, we have $L_1(t)$ is invertible. Then by following the similar steps as in step II in Theorem 1 and the switched system approach, we can conclude that the followers converge to the convex hull formed by the leaders. ∎

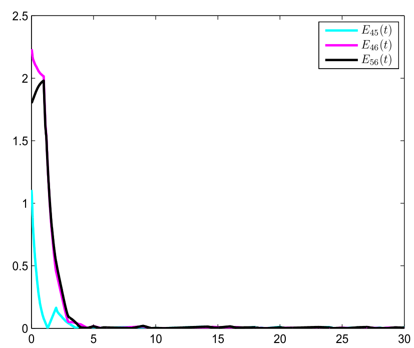


(a)

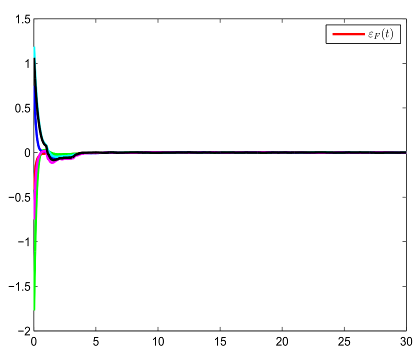


(b)

Fig. 3. Graph topologies of Example 2.



(a)

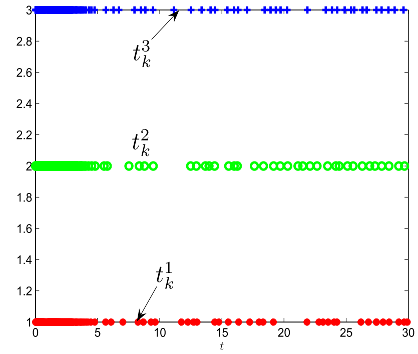


(b)

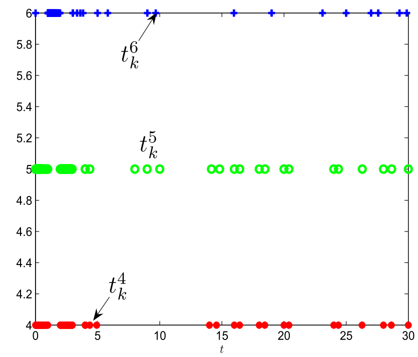
Fig. 4. (a) State trajectories for E_{45}, E_{46} and E_{56} with switching topology; (b) State trajectories for $\varepsilon_{\mathcal{F}}(t)$ with switching topology.

IV. Examples

In this section, we present several simulation results to illustrate the theoretical results derived in this paper.



(a)



(b)

Fig. 5. Event-triggering instants t_k^i with switching topology. (a) $t_k^i, i = 1, 2, 3$; (b) $t_k^i, i = 4, 5, 6$.

Example 1: In this example, we consider the case that the leaders are not opinionated, i.e., $\gamma = 0$ is considered. Assume that the Laplacian matrix has the following form:

$$\begin{bmatrix} 3 & -1 & 0 & -1 & -1 & 0 \\ 0 & 3 & -1 & -1 & -1 & 0 \\ 0 & -1 & 3 & -1 & 0 & -1 \\ 0 & 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 & -1 & 1 \end{bmatrix}.$$

Let $h = 0.25$, $\rho = 0.5$, $\beta = 0.5$, and $\alpha = 0.3$, and the parameters c_1, and c_2 are chosen as $c_1 = 0.02$, $c_2 = 0.03$, respectively. ϱ_i are assumed to be $i/(i + 1)$. The initial positions of the three leaders are, respectively, $[2, 1.5]^T$, $[3, 2]^T$ and $[1, 2.5]^T$ and the other parameters are the same as Example 1. Denote $E_{45} = \|x_4(t) - r_4 - x_5(t) + r_5\|$, $E_{46} = \|x_4(t) - r_4 - x_6(t) + r_6\|$, and $E_{56} = \|x_5(t) - r_5 - x_6(t) + r_6\|$. Fig. 2 shows the state trajectories of E_{45}, E_{46}, E_{56}, and $\varepsilon_{\mathcal{F}}(t)$, which means that the three leaders don't insist on their initial goals and converge to some compromising states and the followers converge to the convex hull formed by the leaders.

Example 2: In this example, the containment control of multi-agent systems with switching topology will be considered. Suppose that the graph is time-varying from the topologies given in Fig. 3. For any time t, if j is a neighbor of i, then $a_{ij}(t) = 1$, otherwise $a_{ij}(t) = 0$. Assume that $T = 2$, $t_{k+1} - t_k = 1$, and the other parameters are the same as the ones in Example 1. Then, Fig. 4 shows the state trajectories of E_{45}, E_{46}, E_{56}, and $\varepsilon_{\mathcal{F}}(t)$ converge to zero exponentially, which confirms our theoretical results well. The event-triggering instants for the leaders' and followers' sets are plotted in Fig. 5.

V. Conclusion

In this paper, the containment control problems of multi-agent networks with fixed and switching topologies have been investigated, respectively, where there exist communication links between the leaders. With a time-dependent triggering function, pull-based event-triggering control protocols have been presented to ensure that the containment control of multi-agent systems with fixed and switching topologies can be solved. Simulations are given to illustrate the effectiveness of our theoretical results. It is interesting to investigate how to generalize the results in this paper into the finite time containment control problem of multi-agent networks [40] or multi-agent systems with time-varying delay [41].

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